Instructions: Legibly complete each of the following on lined paper and submit on Gradescope.

1. Prove congruence modulo n is an equivalence relation for all $n \in \mathbb{Z}_+$.
2. Prove $R = \left\{(a, b) \in \mathbb{Z} \times \mathbb{Z} : a^2 = b^2\right\}$ is an equivalence relation on $\mathbb{Z}$. What are the equivalence classes of this relation?
3. Give an example of a relation which is reflexive and symmetric but not transitive.
4. Prove the function $f\colon \mathbb{Z} \to \mathbb{Z}$ with $f(n) = -4n$ is injective but not surjective.
5. Prove the function $f\colon \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ with $f(q, r) = 8q + r$ is surjective but not injective.
6. Prove the function $f\colon \mathbb{Z} \to \mathbb{N}$ with
$$f(z) = \begin{cases} 2z & \text{if } z \geq 0 \\ -1 - 2z & \text{if } z < 0 \end{cases}$$
is a bijection. Is this weird? Why or why not?